1 CTL and LTL Specifications

1. On the Kripke structure in Figure 1, label the states according to the following specifications:
2. Let $p, q$ be atomic properties of systems. Express the following specifications in CTL as simply as possible: (There can sometimes be several possible solutions.)

(a) $p$ can never happen.
(b) $p$ holds at least twice in the future (i.e., at two different time points).
(c) $p$ cannot hold for two time units.
(d) Whenever $p$ holds, then $q$ can not hold any more.
(e) $p$ holds until $p$ becomes false.
(f) Either $p$ holds in one step, or it will never hold.
(g) If it is possible to reach $p$ at all, then $p$ must be reachable infinitely often.

3. Are the following formulas true, false, or neither?

(a) $(AGp) \rightarrow (AG\neg p)$.
(b) $(AGp) \rightarrow (AG\neg p)$.
(c) $(AFp) \rightarrow (EFp)$.
(d) $(p \land \neg p) \leftrightarrow (q \land \neg p)$.
(e) $(p \land \neg p) \rightarrow \text{false}$.
(f) $(AXp) \rightarrow (EFp)$.
(g) $(AXp) \rightarrow (EF\neg p)$.

4. Represent the following CTL formulas using only $EX$, $EU$, $EG$:

(a) $EF(s \land \neg r)$
(b) $AG(r \rightarrow AFack)$
(c) $AGAFc$
(d) $\text{AGEF}r$

5. For each of the formulas $\phi$ in the last two problems, describe two Kripke structures $K_1, K_2$ with initial states $s_1, s_2$, s.t. $K_1, s_1 \models \phi$ and $K_2, s_2 \not\models \phi$.

6. Show that $AfUg$ is equivalent to

$$\neg[(E(-gU(-g \land \neg f)) \lor EG\neg g].$$

7. Given a formula $EaUb$, and a Kripke structure $K = (S, R, L)$, describe an algorithm which labels all states $s \in S$ where $K, s \models EaUb$, in linear time, i.e., in time $O(|S| + |R|)$.

Note: the algorithm has to label all states $s$ where $K, s \models EaUb$, not just find some such states. For linear time, operations on lists, sets, etc. have to be counted.

8. ** Same as above, for $EGb$.

9. * Let $K_1 = (S_1, R_1, L_1)$ and $K_2 = (S_2, R_2, L_2)$. We define $K_1 \leq K_2$ if $S_1 = S_2$, $R_1 \subseteq R_2$, and $L_1 = L_2$.

   (a) Show that $\leq$ is a partial order.

   (b) Show the following lemma:

   Let $\phi$ be an LTL specification, and $K_1 \leq K_2$ Kripke structures. If $K_2, s \models \phi$ then $K_1, s \models \phi$.

   (c) Show that there exists a CTL specification which cannot be expressed in LTL. Hint: Use the previous Lemma on the formula $EFp$.

10. ** Find a Kripke structure $K, s$ such that $K, s \models AFGp$ but $K, s \not\models AFAGp$.

11. *** Show that exists an LTL specification which cannot be expressed in CTL.

2 Binary Decision Diagrams

1. Describe BDDs for the Boolean functions which are always false and always true.
2. Compute a BDD for the function \((a \lor b) \rightarrow (c \land b)\).

3. Let \(f(x_1, x_2, x_3, x_4)\) be a Boolean function expressing that the number \(x_1x_2x_3x_4\) (in binary notation) is prime, and find a BDD for \(f\).

4. Consider the Boolean function \(f(x_1, \ldots, x_4, y_1, \ldots, y_4)\) which expresses that the binary number \(x_1x_2x_3x_4+1\) equals the binary number \(y_1y_2y_3y_4\).
   
   (a) Describe \(f\) in propositional logic.
   
   (b) Find a BDD for \(f\) with a good variable order.
   
   (c) Generalize the BDD from 4 bits to arbitrary \(n\).

5. Same as above, with \(f\) describing \(x_1x_2x_3x_4 < y_1y_2y_3y_4\).

6. * Can problem 3 (primes) be generalized to arbitrary \(n\)? Why (not)?