Binary Decision Diagrams

1. Describe BDDs for the Boolean functions which are always false and always true.

2. Compute a OBDD $A$ for the function $(a \lor b) \rightarrow (c \land b)$.

3. Compute a OBDD $B$ for the function $(a \land b) \rightarrow d$ with the same variable order as for $A$.

4. For the OBDDs $A$ and $B$, compute the (a) conjunction and (b) disjunction.

5. For the OBDDs $A$ and $B$, compute the OBDD for (a) $A$ NAND $B$ and (b) $\neg A$ AND $B$.

6. Let $f(x_1, x_2, x_3, x_4)$ be a Boolean function expressing that the number $x_1x_2x_3x_4$ (in binary notation) is prime, and find a BDD for $f$. Can the problem be generalized to arbitrary $n$?

7. * Consider the Boolean function $f(x_1, \ldots, x_4, y_1, \ldots, y_4)$ which expresses that the binary number $x_1x_2x_3x_4 + 1$ equals the binary number $y_1y_2y_3y_4$.
   
   (a) Describe $f$ in propositional logic.
   (b) Find a BDD for $f$ with a good variable order.
(c) Generalize the BDD from 4 bits to arbitrary $n$.

8. * Same as above, with $f$ describing $x_1x_2x_3x_4 < y_1y_2y_3y_4$.

9. ** Show that OBDDs are canonical: If OBDD $A$ and OBDD $B$ have the same variable order, and represent the same Boolean function, then they are isomorphic. Hint: Use induction.

10. Let $f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4)$ be the Boolean function from above which expresses that the binary number $x_1x_2x_3x_4+1$ equals the binary number $y_1y_2y_3y_4$, and let $A$ be the BDD for the function $g(x_1, x_2, x_3, x_4)$ where $g(0,0,0,0) = 1$, and $g(x_1, x_2, x_3, x_4) = 0$ otherwise. Compute the BDD for

$$g' := \exists x_1, x_2, x_3, x_4 (f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \land g(x_1, x_2, x_3, x_4))$$

and for

$$g'' := \exists x_1, x_2, x_3, x_4 (f(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \land g'(x_1, x_2, x_3, x_4)).$$

[Hint: there is a simple way to do this!]

11. *5 (optional) Implement a simple OBDD library (in your favorite programming language) which provides

- apply,
- simplify,
- equivalence checking and
- pretty-printing (drawing with the help of some graph drawing software such as dot)